① $4+9+2$

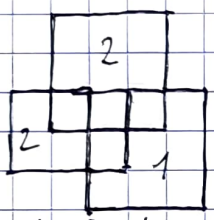
② $5 \times 4 = 20$

③ $27 - 9 = 18$

④ $2a + 2b = 16$
$a + 3b = 14$
$a + b = 8$
$a = 5,\ b = 3$
$a + 2b = \underline{\underline{11}}$

⑤ $6 \times 4 - 1 = 23$

⑥



$5 + 1 + 3 + 4 = 13$

⑦ $1 + 3 + 9 + 27 = 40$

⑧ 1 et 2 : +
3 et 4 : −
5 et 6 : −
3 et 5 : − (3,5) ou (3,6) ou (4,5)
+ 2 essais
→ 6
ou bien
1 et 2 : +
3 et 4 : −
3 et 5 : − → (3,x) ou (4,5)
3 et 6 : − → 4 et 5 : −
+ 6 + (3,4) ou (3,6) ou (4,5)
→ 6

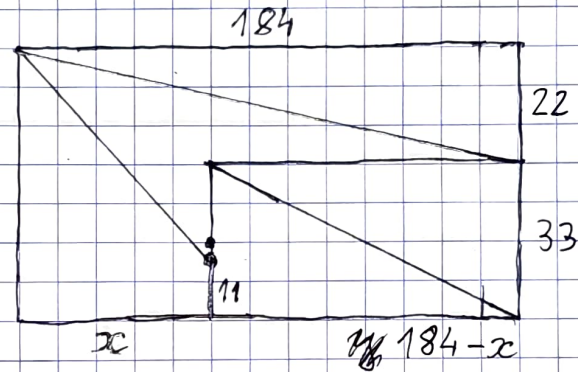
⑨ $A = 3 \rightarrow \geq 1024 \rightarrow$ non

$A = 4 \rightarrow 44 \times 11 + 11 + 11$
$= 46 \times 11 = \underline{\underline{506}}$

$A = 5 \rightarrow$ non

⑩ mod 9 : $0 + 1 + 2 + \dots + 9 \equiv 0 \ [9]$
$2024 \equiv -1 \ [9]$
$0 \equiv (-1) + c \ [9]$
$\rightarrow c = 1$

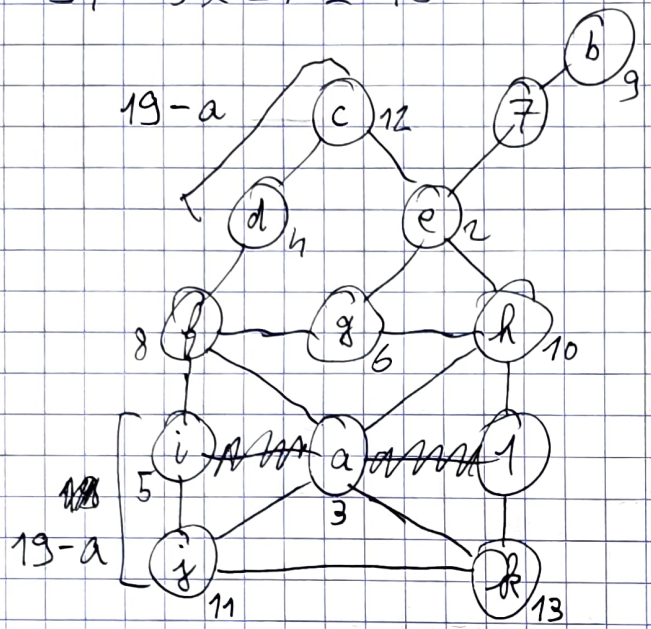
⑪



$184 = 8 \times 23$ $2024 = 8 \times 11 \times 23$
$2024 / 184 = 11 \rightarrow h = 22$
$5 \times 2024 = 184 \times l \rightarrow l = 55$
$2024 / 33 = 8 \times 23 / 3$
$184 - x = 16 \times 23 / 3 = 368 / 3$
$x = 184 - 368/3 \simeq 184 - 123$
$\simeq 61$

(12) $1+2+\dots+13 = 13\times 7 = 91$

$91 - 3\times 24 = 19$



$1+i+k = 19$

$i+k = 18$

$j+k = 24$

$i+j = 19-a$

$b+c+d+e+7 = 19$

$b+c+d+e = 12$

$2+3+4+5$?

$\{j,k\} = \{11,13\}$ $\quad i = 18-k = 5$ avec $k=13$ $\neq 7$

$i = 19-a-j$

$\rightarrow$ $i=5$, $j=11$, $k=13$

$a = 19-i-j = 3$

$f=8$ $\quad h=10$ $\quad g=6$

Reste : 2, 4, 9, 12

$c+d = 16$

$c+e = 14$

$\rightarrow$ $c+d = 12+4$

$c+e = 12+2$

$\rightarrow$ $b=9$

(13) • Si $A+A+1 = X$ (impair)

$R+R \equiv X\ [10]$ imposs.

• Si $A+A = X$ (pair)

$R+R = X+10$ $\quad R = A+5$

$X \leq 8 \quad G \leq 6$

6 9 4 8 5 ← ?
1 9 4 0 3

• Si $A+A+1 = X+10$

$R+R+1 = X$ $\quad R = A-5$

~~reste 0,2,3,5,6,7~~

~~8 9~~
~~1 4 9~~ $X = 9$ imposs. (pas de retenue)

$X = 7$? $\quad G = 6$ reste 0, 2, 4, 5, 7, 9

6 3 8 . .
1 3 8 . . non

(18) 1 10 3 4 5 12 7 16
8
. 9 . .

~~1 7 3 5~~
~~2 4~~

~~1 2 3~~

. 1 10 3 . 5 . 7
8
2 9 4 6

10 1 8 17 6 13 4 15 ~~←~~
11 3 16
9 2 7 12 5 14

10 1 . 3 5 . .
9 2 4

10 1 12 3 8 5 14 17 ←
9 13 16 6
11 2 4 7 15

(14)

$8^2 + 21^2 = 64 + 441 = 505$

$x^2 + y^2 = 505$

$(x+2)^2 + (y-4)^2 = 505$

$x^2 + 4x + 4 + y^2 - 8y + 16 = 505$ 0

$x + 1 - 2y + 4 = 0$

$2y = x + 5$ $\quad x = 2n + 1$

$y = n + 3$

$(2n+1)^2 + (n+3)^2 = 505$

$5n^2 + 10n + 10 = 505$

$n^2 + 2n - 99 = 0$

$\Delta' = 1 + 99 = 100$

$n = -1 \pm 10 \rightarrow n = 9$

$x = 19 \quad y = 12$

$AB = 19 \quad BD = 12 \quad AC = 21 \quad CD = 8$

$19 + 12 + 21 + 8 = 40 + 20 = 60$

(15) $\frac{2}{85} = \frac{1}{m} + \frac{1}{n} = \frac{m+n}{mn}$

$2mn = 85(m+n)$

$m(2n-85) = 85n$

$m = \frac{85n}{2n-85}$

$(85n - 42(2n-85) = n + 42\times85 \quad | \; 85n)$

$n = 43 \quad m = 85\times43 = 3655 \quad s = 3698$

$n = 45 \quad m = 85\times9 = 765 \quad s = 810$ } 3 sol°

$n = 51 \quad m = 85\times3 = 255 \quad s = 306$

$n = 85 \quad m = 85$ impos ($m=n$)

(16) $100k + p + 1 \quad p \mid 100k+1 \quad 10 \le k \le 20$

$k=10$: $p \mid 1001 \quad 1001 = 11\times91 = 7\times11\times13$ → 1012 et 1014

$k=11$: $p \mid 1101 \quad 1101 = 3\times367$ non

$k=12$: 1201 prem.

$k=13$: 1301 prem.

$k=14$: $1401 = 3\times467$

$k=15$: $1501 = 19\times79$ → 1520 et 1580

$k=16$: $1601 =$

$k=17$: $1701 = 9\times189 = 9\times9\times3\times7$

$k=18$: $1801 =$

$k=19$: $1901 =$

$k=20$: ~~2004 $= 8\times11\times23$ → 2022~~

$2001 = 3\times667 = 3\times23\times29$

$31\times7 \quad 37\times3 = 111 \quad 37\times\ldots = \ldots9$

$41\times\ldots = \ldots6 \quad 43\times47 = 2021$ → 4 sol°