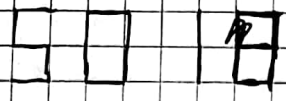
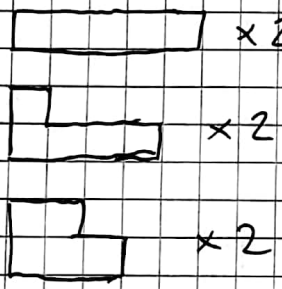
① 

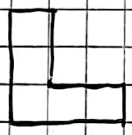
② A = 2 Alice : A, F
B = 4
C = 6 Bob : B, D, E
D = 4
E = 3 Cédric : C, G
F = 9
G = 5
33 / 3 = 11 → Cédric : G

③ 5 + 5 + 5 + 5 + 4 + 4 = 28

④ 2 1
4 7 6
3 5

14 = 1 + 6 + 7
18 = 6 + 7 + 5
14 = 3 + 4 + 7

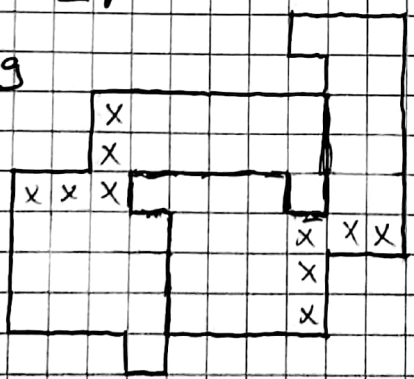
⑤ 



→ 7

⑥ 6 + 7 = 13
3a ≡ 9 [10]
a = 3

⑦ 1 et 2 → 2 façons symétriques
Avec 3 → 3 façons
2 × 3 = 6 façons

⑧ 2 × 29


⑨ n élèves
#options = 2n + 2
= 18 + 22 + 26 = 66
n = 64 / 2 = 32

⑩ C > F
• Si F > E : C > E imp.
→ F < E
2 + jeunes que F ∈ {A, B, D}
Ca ment → C > E > F
• Si A > F : B, D < F < E
A > E B < D < F < E < {A, C}
(2 sol°)
• Si ~~B > F~~ B > F : A, D < F < E
imp. (A)
• Si D > F : A, B < F < E
D > E A > E imp.
→ 2 sol°

(11) $12+13+\dots+20 = 32\times4+16 = 144$

$3S = 144+12+20+x$

$x \equiv 1\ [3]$ $x=13$ ou 16 $S=189/3=63$ $S=64$ $y=63-31-13=19$ imp. $y=64-31-16=17$

12

14 17

18 19

20 13 15 16

$28 = 13+15$

$S=64$

~~11~~, 14, 18

(12) $1+2+3+\dots+15 = \frac{15\times16}{2} = 120$ $S=120/5=24$

~~11~~	6	~~11~~	5	13
14	~~11~~	9	~~11~~	1
2	15	~~11~~	7	~~11~~
~~11~~	3	11	~~11~~	10
8	~~11~~	4	12	~~11~~

$\rightarrow$ 14, 7, 9

$24-15=9=3+6$ ou $4+5$

$24-1=23=10+13$ ou $11+12$

$\binom{3}{6}$ imp $\binom{4}{5}$ imp. $\binom{5}{4}$ imp.

(13) $U>5$ donc $U=7$ ou 8 $N=1$

- $19\times71=1349$ $\dots+1.7.=1349$ $X+F=9$

  $I+7=14$ imp. $(I=7)$

- $19\times81=1539$ $\dots+1.8.=1539$ $X+F=9$

  $I+8=13 \rightarrow I=5$ $.5.+1.8.=1539$

  $D+E+1=5$ $\{D,E\}=\{1,3\}$ imp.

  $\{D,E\}=\{0,4\} \rightarrow D=4,\ E=0$

  $45.+108.=1539$ $\{X,F\}=\{2,7\}$

  $\rightarrow$ 4082 et 4087

(14) Pas de retenues.

$102^2 = 10404$

$201^2 = 40401$

$103^2 = 10609$

$301^2 = 90601$

$112^2 = 12544$

$211^2 = 44521$

$113^2 = 12769$

$311^2 = 96721$

$122^2 = 14884$

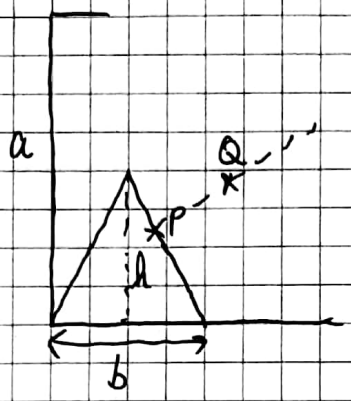
$221^2 = 48841$

10 sol°?

(15) 1s : $\frac{33}{60}$ tour = $\frac{11}{20}$

20s → 11 tours → $\underline{\underline{20}}$

(17)



$h = b\frac{\sqrt{3}}{2}$

$P_x = a - h = a - b\frac{\sqrt{3}}{2}$

$P_y = \frac{b}{2}$

$4\sqrt{3} \approx 6{,}928$

$7 - 4\sqrt{3} \approx 0{,}072$

$\times 400 = 28{,}8$

$\rightarrow$ 28,8

$$\frac{h}{b/2} = \frac{P_y}{b - P_x} \rightarrow \frac{b\sqrt{3}/2}{b(1+\frac{\sqrt{3}}{2}) - a} = \sqrt{3}$$

$b/2 = b(\sqrt{3} + \frac{3}{2}) - a\sqrt{3}$

$a\sqrt{3} = b(\sqrt{3} + 1) \rightarrow b = \frac{\sqrt{3}}{1+\sqrt{3}}\, a$

$$P_x = a\left(1 - \frac{3}{2(1+\sqrt{3})}\right) = \frac{2+2\sqrt{3}-3}{2(1+\sqrt{3})}\, a$$

$$= \frac{2\sqrt{3}-1}{2(1+\sqrt{3})}\, a = \frac{(2\sqrt{3}-1)(\sqrt{3}-1)}{4}\, a = \frac{7-3\sqrt{3}}{4}\, a$$

$$P_y = \frac{\sqrt{3}}{2(1+\sqrt{3})}\, a = \frac{3-\sqrt{3}}{4}\, a$$

$$Q_x = a - P_y = \frac{4-3+\sqrt{3}}{4}\, a = \frac{1+\sqrt{3}}{4}\, a$$

$$Q_y = P_x = \frac{7-3\sqrt{3}}{4}\, a$$

$$Q_x - P_x = \frac{-6+4\sqrt{3}}{4}\, a \qquad (Q_x - P_x)\cdot 400 = \frac{-3+2\sqrt{3}}{2}\, a$$

$$(Q_x - P_x)^2 = \frac{9+12-12\sqrt{3}}{4}\, a^2 = \frac{21-12\sqrt{3}}{4}\, a^2$$

$$Q_y - P_y = \frac{4-2\sqrt{3}}{4}\, a = \frac{2-\sqrt{3}}{2}\, a$$

$$(Q_y - P_y)^2 = \frac{7-4\sqrt{3}}{4}\, a^2$$

$$\rightarrow \frac{28-16\sqrt{3}}{4}\, a^2 = (7-4\sqrt{3})a^2$$

(18) $a^3+c^3 \equiv 0\ [19]$

$c \equiv -a\ [19]$?

0
1
8
27 ≡ 8

$b > c$ $\quad b = c+1$?

$(b=c)$

1
8
27
64
125

$20(a^3+c^3) = 19(a^3+b^3)$

$20 \mid a^3+b^3$ ~~a et b même parité~~

$b \equiv -a\ [5]$ $\quad b \equiv -a\ [4]$

$b \equiv -a\ [20]$

a, $b = 20-a$, $c = 19-a$ ~~non bon~~

$$\frac{a^3+b^3}{a^3+c^3} = 1 + \frac{b^3-c^3}{a^3+c^3} = 1 + \frac{(c+1)^3-c^3}{(19-c)^3+c^3}$$

~~1 19 a~~ $(19-c)^3 + c^3 = 19\left((c+1)^3 - c^3\right)$

$19^3 - 3\times 19^2 c + 3\times 19 c^2 = 19(3c^2+3c+1)$

$19^2 - 3\times 19 c + 3c^2 = 3c^2 + 3c + 1$

$60c = 19^2 - 1 = 20\times 18 \rightarrow c = 6,\ a = 13,\ b = 7$

(13, 7, 6)
(26, 14, 12)
(39, 21, 18)

(18) suite

a, $b = 40 - a$, $c = 38 - a$ $\quad (b = c + 2)$

$(38 - c)^3 + c^3 = 19\left((c+2)^3 - c^3\right)$

$38^3 - 3 \times 38^2 c + 3 \times 38 c^2 = 19(6c^2 + 12c + 8)$

$8 \times 19^2 - 12 \times 19 c + \cancel{6c^2} = \cancel{6c^2} + 12c + 8$

$240c = 8(19^2 - 1)$

$30c = 120 \times 18$

$c = 2 \times 6 = 12$

$b = 14$

$a = 26$ déjà trouvée.

(16)

$$\begin{array}{r} 4038 \\ 12114 \\ \quad 8076 \\ \quad 12114 \\ \hline 53422\,74 \end{array}$$