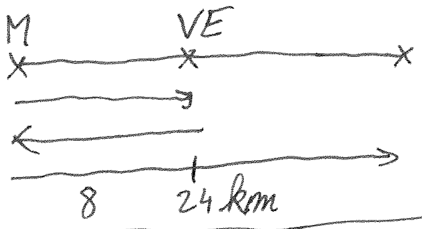
1



$\boxed{16\ km}$

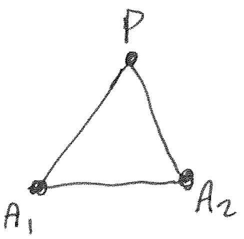
2

0 : 1
1 : 2
2 : 4
3 : 8
4 : 16
5 : 32

6 : 50
7 : 68
8 : 86
9 : 104
10 : 122

$\longrightarrow \boxed{121}$

3



$75° \quad - \quad 30k°$

$\downarrow$

30, 60, 90

~~105 - 30k~~

75	30	75	$\rightarrow 3\times 2$
75	60	45	$\rightarrow 6\times 2$
75	90	15	$\rightarrow 6\times 2$
			$\boxed{30}$

4

~~482~~

$48 = 2^4 . 3 =$

$1512 = 9\times 3^3 \times 7 \times 2^3$

168

56

$840 = 5 \times 7 \times 2^3 \times 3$

120

~~24~~

B C D

$1080 = 5 \times 9 \times$

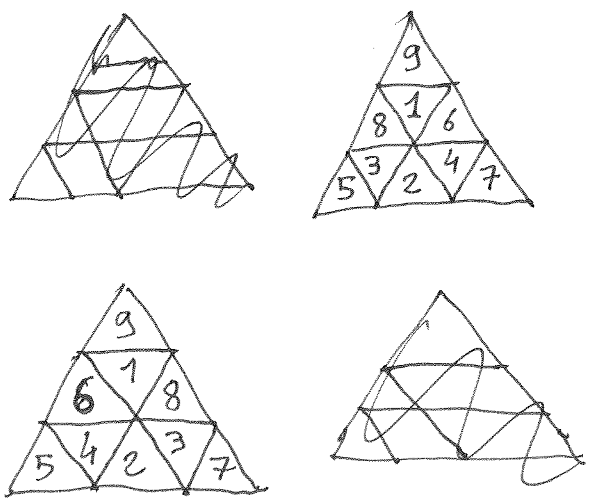
120

24

$840 \rightarrow 2^3 . 3$

$1080 \rightarrow 2^3 . 3$

$1512 \rightarrow 2^3 . 3$



$24 = 1 \times 3 \times 8$

$= 1 \times 4 \times 6$

$= 2 \times 3 \times 4$

$\rightarrow 1, 3, 4$

$\boxed{6 \text{ solutions}}$

5

$\boxed{1}\ \boxed{6}\ 2 \times \boxed{4}\ \boxed{3} = 6\ 9\ 6\ 6$

$+ \quad \boxed{\cancel{89}}\ n\ \boxed{\cancel{82}} + \boxed{1}\ p\ \boxed{6} = \boxed{1}\ \boxed{\cancel{0}}\ \boxed{8}\ q$

$\boxed{2}\ 5\ \boxed{\cancel{4}} \times \boxed{2}\ \boxed{7} = \boxed{6}\ \boxed{8}\ 5\ \boxed{8}$ $\quad \boxed{6858}$

$\cancel{6}, 7, \cancel{8}$

$A = 0$ ou 1

$6966 = 9 \times 774 = 2 \times 9 \times 387 = 2 \times 3^4 \times 43$

$\square\square 2 = 2 \times 81 \quad - 43$ $\qquad 2 \equiv 2 \times 1$

$$\begin{array}{r} 162 \\ \times\ 43 \\ \hline 6966 \end{array}$$

$n + p = q \qquad n \geqslant 88 \qquad 107 \leqslant q \leqslant 116$

$p = q - n \leqslant 116 - 88 = 28 \qquad p \leqslant 28$

$685x \mid 25y \qquad 6850 \mid 250 \qquad 137 \mid 5$

$6860 / 250 =$

$\to 675/25 = 135/5 = \cancel{27} \to 28$ (max)

$\cancel{6850/260}$ $\quad 685/26$

$\cancel{342/343/13}$ $\quad 342/13 \to 26$ (min)

$6850 \mid 26$
$165 \mid 25$

169×4

$\cancel{250 \times}$

$260 \times 26 \leqslant 170 \times 40 \leqslant 6800$

$\cancel{6850 \mid 27}$ $\qquad \underbrace{685}_{19+8}8$

$250 \times 28 = 7000$

$$\begin{array}{r} 254 \\ \times\ 27 \\ \hline 6858 \end{array}$$

6

9 - 7 - 7 - 9 $\boxed{19}$

7

~~A : -19 à 3~~
~~B :~~

$$a+b+c+d+e+f=0$$
$$a+c+d+e+f=2 \quad \Big) \Rightarrow b = -2$$

$$c+d+e+f=18 \Rightarrow a = -16$$

$$a+d+e+f=3 \Rightarrow d+e+f=19 \Rightarrow c=-1$$

$$e+f=17 \Rightarrow d=2$$

~~a+b+c+d+e~~ $a+d+f=-3 \Rightarrow f=11$

$\Rightarrow e = \cancel{16}$ 6

$P = +2 \times -16 \times 2 \times 6 \times 11 \cancel{=}$ $\boxed{-4224}$

64 704
4224

$$\begin{array}{r} 176 \\ \times\ 24 \\ \hline 4224 \end{array}$$

8

~~Noyaux~~
~~k(n+1)~~ $k(n+1)+1 = 1993$

$n+1 \mid 1992$ $\qquad 1992 = 2^3 . 3 . 83$

249
83

$$n+1 = 83 \times \begin{cases} 1 \\ 2 \\ 3 \end{cases}$$

$\boxed{n = 82,\ 165 \text{ ou } 248}$: 3 solutions.

9

x^2 —

84

$x^2 - 336 = y^2$

$y^2 - 336 = z^2$

$x^2 - y^2 = y^2 - z^2$

m̂ parité ⇒ pairs.

$\alpha^2 - \beta^2 = 336 = (\alpha-\beta)(\alpha+\beta)$ α et β : m̂ parité

$\ \ \|\qquad\qquad \cancel{\ }\,4\qquad\ \ 4$

$2^4 \times 3 \times 7$ ← pairs.

3 7 → $\begin{cases}\alpha = 4.5\\ \beta = 4.2\end{cases}$ ou $\begin{cases}\alpha = 4.11\\ \beta = 4.10\end{cases}$

1 21

$\boxed{20 \text{ et } 8}$ $\boxed{44 \text{ et } 40}$

$400 - 64 = 336$ $44^2 - 40^2 = 16\,(11^2 - 10^2) = 16 \times 21$

impairs : $\alpha = 2\alpha' + 1$, $\beta = 2\beta' + 1$

$4(\alpha' - \beta')(\alpha' + \beta' + 1)$ $(\alpha' - \beta')(\alpha' + \beta' + 1) = 2 \times 2 \times 3 \times 7$

4 21 → ~~8 et 11~~ 8 et 12 → 17 et 25

12 7 → 2 et 9 → 5 et 19 ×

28 3 → 12 et 15 → 25 et 31

84 1 → 41 et 42 → 83 et 85 ×

$31 \to 25 \to 17$ $\boxed{289}$

$961 \to 625 \to 289$

$\cancel{336}$ 336

961 625

10

$a + b = ab$

$(a-1)(b-1) = 1$

$a - 1 = 2^{k} \cdot 5^{l}$ avec $|k| \leq 3$ et $|l| \leq 3$

$b - 1 = 2^{-k} \cdot 5^{-l}$

$\frac{1}{16}$

~~Si $a-1<0, b-1<0$~~

$a \geq 0 \Rightarrow a - 1 \geq -1$

$b - 1 \geq -1$

$(0;0)$, $(k;l)$ $\rightarrow$ $\boxed{50}$

$\downarrow \downarrow$

$7 \quad 7$

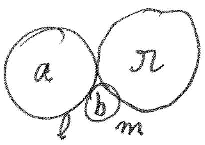
11

ab carré

ar "

br "

$3 \leq r \leq 100$



$a \wedge r = 1$

$$\begin{cases} (a+b)^2 = l^2 + (a-b)^2 \\ (b+r)^2 = m^2 + (r-b)^2 \\ (a+r)^2 = (m+l)^2 + (r-a)^2 \end{cases}$$

$$b = \frac{ar(a+r) \pm 2ar\sqrt{ar}}{(r-a)^2}$$

$l^2 = 4ab$

$m^2 = 4br$

$(m+l)^2 = 4ar = 4ab + 4br + 2ml$

~~4(ar~~ $16(ar - ab - br)^2 = 4 \times 16\, ab^2 r$

$(ar - ab - br)^2 = 4ab^2 r \longrightarrow ar$: carré pf.

$a^2 r^2 + (a+r)^2 b^2 - 2ar(a+r)b = 4rb^2$

$(r-a)^2 b^2 - 2ar(a+r)b + a^2 r^2 = 0$

$\Delta' = a^2 r^2 (a+r)^2 - a^2 r^2 (r-a)^2 = 4a^3 r^3$

11

$$(a-b)^2 r^2 - 2(a-b)abr + a^2b^2 = 4ab^2 r$$

$$(a-b)^2 r^2 - 2(a+b)abr + a^2b^2 = 0$$

$$\Delta' = a^2b^2(a+b)^2 - a^2b^2(a-b)^2$$
$$= 4a^2b^2ab = 4a^3b^3$$

$$r = \frac{(a+b)ab \pm 2ab\sqrt{ab}}{(a-b)^2} = \frac{ab}{(a-b)^2}\left[a+b \pm 2\sqrt{ab}\right]$$

$$\cancel{b=1,\ a=k^2 \quad \frac{k^2}{(k^2-1)^2}\left[k^2+1 \pm 2k\right]}$$

$$a+b=k \qquad \sqrt{ab}=l \qquad 2l \leq k$$

$$\frac{l^2}{k^2-4l^2}\left[k \pm 2l\right] = \frac{l^2(k \pm 2l)}{(k-2l)(k+2l)} = \frac{l^2}{k \pm 2l} = r$$

$$\cancel{\Rightarrow \boxed{k \mid l} \quad a^2+b^2 \mid ab} \qquad k \pm 2l \mid l^2$$

$$\frac{l^2}{k \pm 2l} \geq l \qquad kl \pm 2l^2 \leq l^2$$

$$\frac{l^2}{k+2l} \geq l \Rightarrow l^2 \geq kl + 2l^2 \Rightarrow \text{contr.}$$

$$\boxed{r = \frac{l^2}{k-2l}} \qquad \frac{l^2}{k-2l} \geq l \Rightarrow kl \leq 3l^2 \qquad k \leq 3l$$

$$\boxed{2l \leq k \leq 3l} \qquad l^2 \leq 100(k-2l)$$

$$\frac{l^2}{k-2l} \geq \frac{k}{2} \qquad l^2 \geq \frac{k^2}{2} - kl$$

$$k^2 \leq 2l(k+l)$$

$$\frac{k}{l} \leq 2\left(1 + \frac{l}{k}\right)$$

11

$ab = l^2$
$a+b = k$

$x^2 - kx + l^2 = 0$

$\Delta = k^2 - 4l^2$ carrée.

$k \wedge l = 1$

$l = 2 \quad k = 5 : \quad 4, 1, 4$

$l = 3 \quad k = 10$

$l = 4 \quad k = 10 \qquad r = \frac{16}{2} = 8 \qquad (\cancel{a = 8})$

$l = 5$ /

$l = 6 : \quad ab = 2 \times 2 \times 3 \times 3 \qquad b = 4,\ a = 9,\ k = 13$

$r = 36/1 = 36$

$4, 9, \boxed{36} \qquad r = ab$

$$\frac{l^2}{k - 2l} = l^2 \qquad \boxed{36}$$

$k - 2l = 1$

$a + b - 2\sqrt{ab} = 1$

$4ab = (a + b - 1)^2$

$a^2 + b^2 - 2ab - 2a - 2b + 1 = 0$

$n^2 + p^2 - 2np = 1$

$p \geq 1 \quad \Delta' = n^2 - (n^2 - 1) = 1$

$(n - p)^2 = 1$